FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Monday 29th January, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P, then the common ratio of the G.P. is equal to

(1) 7 (2) 4

(3) 5 (4) 6

Ans. (4)

Sol. $a+ar+ar^2+ar^3+....+ar^{63}$

$=7(a+ar^2+ar^4.....+ar^{62})$

$$\Rightarrow \frac{a(1-r^{64})}{1-r}=\frac{7a(1-r^{64})}{1-r^2}$$

$r=6$

2. In an A.P., the sixth terms $a_6 = 2$. If the $a_1a_4a_5$ is the greatest, then the common difference of the A.P., is equal to

(1) $\frac{3}{2}$ (2) $\frac{8}{5}$ (3) $\frac{2}{3}$ (4) $\frac{5}{8}$

Ans. (2)

Sol. $a_6=2 \Rightarrow a+5d=2$

$a_1a_4a_5=a(a+3d)(a+4d)$

$=(2-5d)(2-2d)(2-d)$

$f(d)=8-32d+34d^2-20d+30d^2-10d^3$

$f'(d)=-2(5d-8)(3d-2)$

$-$ $+$ $-$

2/3 8/5

$d=\frac{8}{5}$

3. If $f(x)=\begin{cases}2+2x, & -1\le x<0\\ 1-\frac{x}{3}, & 0\le x\le 3\end{cases}$; $g(x)=\begin{cases}-x, & -3\le x\le 0\\ x, & 0<x\le 1\end{cases}$,

then range of (fog(x)) is

(1) (0, 1] (2) [0, 3)

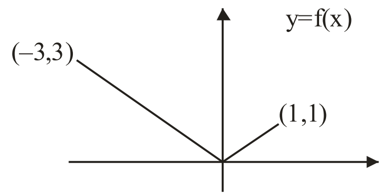
(3) [0, 1] (4) [0, 1)

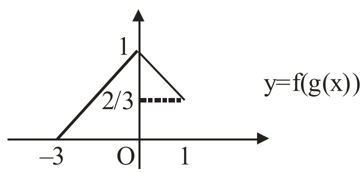
Ans. (3)

Sol. $$f(g(x))=\begin{cases}2+2g(x), & -1\le g(x)<0 \quad .....(1)\\ 1-\frac{g(x)}{3}, & 0\le g(x)\le 3 \quad .....(2)\end{cases}$$

By (1) $x\in\phi$

And by (2) $x\in[-3,0]$ and $x\in[0,1]$





Range of $f(g(x))$ is [0, 1]

4. A fair die is thrown until 2 appears. Then the probability, that 2 appears in even number of throws , is

(1) $\frac{5}{6}$ (2) $\frac{1}{6}$ (3) $\frac{5}{11}$ (4) $\frac{6}{11}$

Ans. (3)

Sol. Required probability =

$$\frac{5}{6}\times\frac{1}{6}+\left(\frac{5}{6}\right)^3\times\frac{1}{6}+\left(\frac{5}{6}\right)^5\times\frac{1}{6}+.....$$

$$=\frac{1}{6}\times\frac{\frac{5}{6}}{1-\frac{25}{36}}=\frac{5}{11}$$

5. If $z=\frac{1}{2}-2i$, is such that

$|z+1|=\alpha z+\beta(1+i), i=\sqrt{-1}$ and $\alpha,\beta\in R$, then $\alpha+\beta$ is equal to

(1) –4 (2) 3

(3) 2 (4) –1

Ans. (2)

Sol. $z=\frac{1}{2}-2i$

$$|z+1|=\alpha z+\beta(1+i)$$

$$\left|\frac{3}{2}-2i\right|=\frac{\alpha}{2}-2\alpha i+\beta+\beta i$$

$$\left|\frac{3}{2}-2i\right|=\left(\frac{\alpha}{2}+\beta\right)+(\beta-2\alpha)i$$

$$\beta=2\alpha \text{ and } \frac{\alpha}{2}+\beta=\sqrt{\frac{9}{4}+4}$$

$$\alpha+\beta=3$$

6. $\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{1}{\left(x-\frac{\pi}{2}\right)^2}\int_{x^3}^{\left(\frac{\pi}{2}\right)^3}\cos\left(\frac{1}{t^3}\right)dt\right)$ is equal to

(1) $\frac{3\pi}{8}$ (2) $\frac{3\pi^2}{4}$

(3) $\frac{3\pi^2}{8}$ (4) $\frac{3\pi}{4}$

Ans. (3)

Sol. Using L'hopital rule

$$=\lim_{x\to\frac{\pi^-}{2}}\frac{0-\cos x\times 3x^2}{2\left(x-\frac{\pi}{2}\right)}$$

$$=\lim_{x\to\frac{\pi^-}{2}}\frac{\sin\left(x-\frac{\pi}{2}\right)}{2\left(x-\frac{\pi}{2}\right)}\times\frac{3\pi^2}{4}$$

$$=\frac{3\pi^2}{8}$$

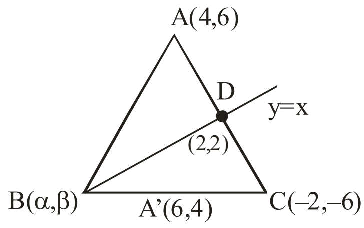
7. In a ΔABC, suppose $y=x$ is the equation of the bisector of the angle B and the equation of the side AC is $2x-y=2$. If 2AB = BC and the point A and B are respectively (4, 6) and (α,β), then $\alpha+2\beta$ is equal to

(1) 42 (2) 39

(3) 48 (4) 45

Ans. (1)

Sol.



AD : DC = 1 : 2

$$\frac{4-\alpha}{6-\alpha}=\frac{10}{8}$$

$$\alpha=\beta$$

$$\alpha=14 \text{ and } \beta=14$$

8. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear .if $\vec{a}+5\vec{b}$ is collinear with $\vec{c},\vec{b}+6\vec{c}$ is collinear with $\vec{a}$ and $\vec{a}+\alpha\vec{b}+\beta\vec{c}=\vec{0}$, then $\alpha+\beta$ is equal to

(1) 35 (2) 30

(3) – 30 (4)–25

Ans. (1)

Sol. $\vec{a}+5\vec{b}=\lambda\vec{c}$

$\vec{b}+6\vec{c}=\mu\vec{a}$

Eliminating $\vec{a}$

$$\lambda\vec{c}-5\vec{b}=\frac{6}{\mu}\vec{c}+\frac{1}{\mu}\vec{b}$$

$$\therefore \mu=\frac{-1}{5},\lambda=-30$$

$$\alpha=5,\beta=30$$

9. Let $\left(5, \frac{a}{4}\right)$, be the circumcenter of a triangle with vertices $A(a,-2)$, $B(a, 6)$ and $C\left(\frac{a}{4},-2\right)$. Let α denote the circumradius, β denote the area and γ denote the perimeter of the triangle. Then $\alpha+\beta+\gamma$ is

(1) 60 (2) 53

(3) 62 (4) 30

Ans. (2)

Sol. $A(a, -2), B(a, 6), C\left(\frac{a}{4},-2\right)$, $O\left(5,\frac{a}{4}\right)$

AO = BO

$$(a-5)^2+\left(\frac{a}{4}+2\right)^2=(a-5)^2+\left(\frac{a}{4}-6\right)^2$$

a = 8

AB = 8, AC = 6, BC = 10

$\alpha=5, \beta=24, \gamma=24$

10. For $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, if

$$y(x)=\int \frac{\operatorname{cosec} x+\sin x}{\operatorname{cosec} x \sec x+\tan x \sin^2 x}dx \text{ and}$$

$\lim\limits_{x\to\left(\frac{\pi}{2}\right)^-} y(x)=0$ then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (2) $\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(3) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (4) $\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$

Ans. (4)

Sol. $$y(x)=\int\frac{(1+\sin^2 x)\cos x}{1+\sin^4 x}dx$$

Put sinx = t

$$=\int\frac{1+t^2}{t^4+1}dt = \frac{1}{\sqrt{2}}\tan^{-1}\frac{\left(t-\frac{1}{t}\right)}{\sqrt{2}}+C$$

$x=\frac{\pi}{2}, t=1$ $\quad\therefore C=0$

$$y\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$$

11. If $\alpha, -\frac{\pi}{2}<\alpha<\frac{\pi}{2}$ is the solution of $4\cos\theta+5\sin\theta=1$, then the value of $\tan\alpha$ is

(1) $\frac{10-\sqrt{10}}{6}$ (2) $\frac{10-\sqrt{10}}{12}$

(3) $\frac{\sqrt{10}-10}{12}$ (4) $\frac{\sqrt{10}-10}{6}$

Ans. (3)

Sol. $4+5\tan\theta=\sec\theta$

Squaring : $24\tan^2\theta+40\tan\theta+15=0$

$$\tan\theta=\frac{-10\pm\sqrt{10}}{12}$$

and $\tan\theta=-\left(\frac{10+\sqrt{10}}{12}\right)$ is Rejected.

(3) is correct.

12. A function y = f(x) satisfies

$f(x)\sin 2x+\sin x-(1+\cos^2 x)f'(x)=0$ with condition

f(0) = 0 . Then $f\left(\frac{\pi}{2}\right)$ is equal to

(1) 1 (2) 0 (3) –1 (4) 2

Ans. (1)

Sol. $$\frac{dy}{dx}-\left(\frac{\sin 2x}{1+\cos^2 x}\right)y=\sin x$$

I.F. = $1 + \cos^2x$

$$y\cdot(1+\cos^2 x)=\int(\sin x)dx$$

= – cosx + C

x = 0, C = 1

$$y\left(\frac{\pi}{2}\right)=1$$

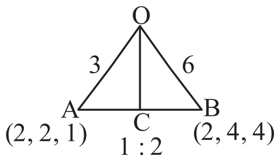
13. Let O be the origin and the position vector of A and B be $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$ respectively. If the internal bisector of $\angle AOB$ meets the line AB at C, then the length of OC is

(1) $\frac{2}{3}\sqrt{31}$ (2) $\frac{2}{3}\sqrt{34}$

(3) $\frac{3}{4}\sqrt{34}$ (4) $\frac{3}{2}\sqrt{31}$

Ans. (2)

Sol.



length of $OC = \frac{\sqrt{136}}{3} = \frac{2\sqrt{34}}{3}$

14. Consider the function $f:\left[\frac{1}{2},1\right] \to R$ defined by $f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$. Consider the statements

(I) The curve y = f(x) intersects the x-axis exactly at one point

(II) The curve y = f(x) intersects the x-axis at $x = \cos\frac{\pi}{12}$

Then

(1) Only (II) is correct

(2) Both (I) and (II) are incorrect

(3) Only (I) is correct

(4) Both (I) and (II) are correct

Ans. (4)

Sol. $f'(x) = 12\sqrt{2}x^2 - 3\sqrt{2} \geq 0$ for $\left[\frac{1}{2},1\right]$

$f\left(\frac{1}{2}\right) < 0$

$f(1) > 0 \Rightarrow$ (A) is correct.

$f(x) = \sqrt{2}(4x^3 - 3x) - 1 = 0$

Let $\cos\alpha = x$,

$\cos 3\alpha = \cos\frac{\pi}{4} \Rightarrow \alpha = \frac{\pi}{12}$

$x = \cos\frac{\pi}{12}$

(4) is correct.

15. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ and $|2A|^3 = 2^{21}$ where $\alpha, \beta \in Z$,

Then a value of α is

(1) 3 (2) 5

(3) 17 (4) 9

Ans. (2)

Sol. $|A| = \alpha^2 - \beta^2$

$|2A|^3 = 2^{21} \Rightarrow |A| = 2^4$

$\alpha^2 - \beta^2 = 16$

$(\alpha + \beta)(\alpha - \beta) = 16 \Rightarrow \alpha = 4$ or 5

16. Let PQR be a triangle with $R(-1, 4, 2)$. Suppose M(2, 1, 2) is the mid point of PQ. The distance of the centroid of ΔPQR from the point of intersection of the line $\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$ and $\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$ is

(1) 69 (2) 9

(3) $\sqrt{69}$ (4) $\sqrt{99}$

Ans. (3)

Sol. Centroid G divides MR in 1 : 2

G(1, 2, 2)

Point of intersection A of given lines is (2, –6, 0)

$AG = \sqrt{69}$

17. Let R be a relation on Z × Z defined by

(a, b)R(c, d) if and only if ad – bc is divisible by 5.

Then R is

(1) Reflexive and symmetric but not transitive

(2) Reflexive but neither symmetric not transitive

(3) Reflexive, symmetric and transitive

(4) Reflexive and transitive but not symmetric

Ans. (1)

Sol. (a, b)R(a, b) as ab – ab = 0

Therefore reflexive

Let (a,b)R(c,d)⇒ad – bc is divisible by 5

⇒ bc – ad is divisible by 5 ⇒(c,d)R(a,b)

Therefore symmetric

Relation not transitive as (3,1)R(10,5) and (10,5)R(1,1) but (3,1) is not related to (1,1)

18. If the value of the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin x^{2023}}}\right)dx=\frac{\pi}{4}(\pi+a)-2,$$

then the value of a is

(1) 3 (2) $-\frac{3}{2}$ (3) 2 (4) $\frac{3}{2}$

Ans. (1)

Sol. $I=\int_{-\pi/2}^{\pi/2}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin x^{2023}}}\right)dx$

$I=\int_{-\pi/2}^{\pi/2}\left(\frac{x^2\cos x}{1+\pi^{-x}}+\frac{1+\sin^2 x}{1+e^{\sin(-x)^{2023}}}\right)dx$

On Adding, we get

$$2I=\int_{-\pi/2}^{\pi/2}\left(x^2\cos x+1+\sin^2 x\right)dx$$

On solving

$$I=\frac{\pi^2}{4}+\frac{3\pi}{4}-2$$

a = 3

19. Suppose

$$f(x)=\frac{\left(2^x+2^{-x}\right)\tan x\sqrt{\tan^{-1}\left(x^2-x+1\right)}}{\left(7x^2+3x+1\right)^3},$$

Then the value of f '(0) is equal to

(1) π (2) 0

(3) $\sqrt{\pi}$ (4) $\frac{\pi}{2}$

Ans. (3)

Sol. $f'(0)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}$

$=\lim_{h\to 0}\frac{(2^h+2^{-h})\tan h\sqrt{\tan^{-1}(h^2-h+1)}-0}{(7h^2+3h+1)^3 h}$

$=\sqrt{\pi}$

20. Let A be a square matrix such that $AA^T=I$. Then $\frac{1}{2}A\left[\left(A+A^T\right)^2+\left(A-A^T\right)^2\right]$ is equal to

(1) A^2+I (2) A^3+I

(3) A^2+A^T (4) A^3+A^T

Ans. (4)

Sol. $AA^T=I=A^TA$

On solving given expression, we get

$\frac{1}{2}A\left[A^2+(A^T)^2+2AA^T+A^2+(A^T)^2-2AA^T\right]$

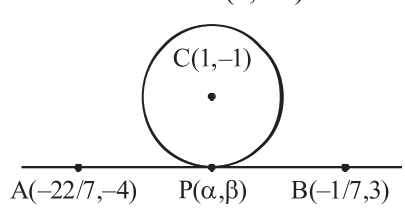
$=A[A^2+(A^T)^2]=A^3+A^T$

SECTION-B

21. Equation of two diameters of a circle are $2x-3y=5$ and $3x-4y=7$. The line joining the points $\left(-\frac{22}{7},-4\right)$ and $\left(-\frac{1}{7},3\right)$ intersects the circle at only one point $P(\alpha,\beta)$. Then $17\beta-\alpha$ is equal to

Ans. (2)

Sol. Centre of circle is (1, –1)



Equation of AB is 7x – 3y + 10 = 0 …(i)

Equation of CP is 3x + 7y + 4 = 0 …(ii)

Solving (i) and (ii)

$\alpha=\frac{-41}{29},\beta=\frac{1}{29}$ $\therefore 17\beta-\alpha=2$

22. All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" IS

Ans. (553)

Sol. Words starting with E = 360

Words starting with GE = 60

Words starting with GN = 60

Words starting with GTE = 24

Words starting with GTN = 24

Words starting with GTT = 24

GTWENTY = 1

Total = 553

23. Let α,β be the roots of the equation $x^2-x+2=0$ with $\mathrm{Im}(\alpha)>\mathrm{Im}(\beta)$. Then $\alpha^6+\alpha^4+\beta^4-5\alpha^2$ is equal to

Ans. (13)

Sol. $\alpha^6+\alpha^4+\beta^4-5\alpha^2$

$= \alpha^4(\alpha-2)+\alpha^4-5\alpha^2+(\beta-2)^2$

$= \alpha^5-\alpha^4-5\alpha^2+\beta^2-4\beta+4$

$= \alpha^3(\alpha-2)-\alpha^4-5\alpha^2+\beta-2-4\beta+4$

$= -2\alpha^3-5\alpha^2-3\beta+2$

$= -2\alpha(\alpha-2)-5\alpha^2-3\beta+2$

$= -7\alpha^2+4\alpha-3\beta+2$

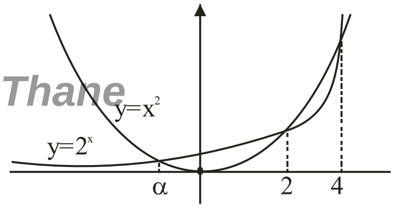
$= -7(\alpha-2)+4\alpha-3\beta+2$

$= -3\alpha-3\beta+16=-3(1)+16=13$

24. Let $f(x)=2^x-x^2, x\in R$. If m and n are respectively the number of points at which the curves y = f(x) and y = f '(x) intersects the x-axis, then the value of m + n is

Ans. (5)

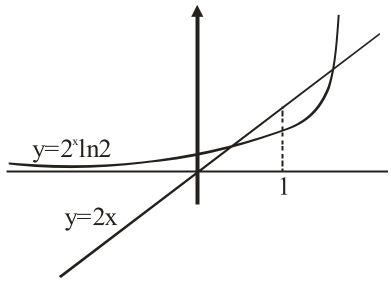
Sol.



$\therefore$ m = 3

$f'(x)=2^x\ln 2-2x=0$

$2^x\ln 2=2x$



$\therefore$ n = 2

$\Rightarrow$ m + n = 5

25. If the points of intersection of two distinct conics $x^2+y^2=4b$ and $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ lie on the curve $y^2=3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is __

Ans. (432)

Sol. Putting $y^2=3x^2$ in both the conics

We get $x^2=b$ and $\frac{b}{16}+\frac{3}{b}=1$

$\Rightarrow b=4,12$ (b = 4 is rejected because curves coincide)

$\therefore$ b = 12

Hence points of intersection are

$\left(\pm\sqrt{12},\pm 6\right)$ $\Rightarrow$ area of rectangle = 432

26. If the solution curve $y = y\ x$ of the differential equation $\left(1+y^2\right)\left(1+\log_e x\right)dx + x\,dy = 0,\ x > 0$ passes through the point (1, 1) and $y(e) = \dfrac{\alpha - \tan\left(\frac{3}{2}\right)}{\beta + \tan\left(\frac{3}{2}\right)}$, then $\alpha + 2\beta$ is

Ans. (3)

Sol. $\displaystyle\int\left(\frac{1}{x}+\frac{\ln x}{x}\right)dx+\int\frac{dy}{1+y^2}=0$

$$\ln x + \frac{(\ln x)^2}{2} + \tan^{-1} y = C$$

Put x = y = 1

$$\therefore C = \frac{\pi}{4}$$

$$\Rightarrow \ln x + \frac{(\ln x)^2}{2} + \tan^{-1} y = \frac{\pi}{4}$$

Put x = e

$$\Rightarrow y = \tan\left(\frac{\pi}{4}-\frac{3}{2}\right) = \frac{1-\tan\frac{3}{2}}{1+\tan\frac{3}{2}}$$

$$\therefore \alpha = 1, \beta = 1$$

$$\Rightarrow \alpha + 2\beta = 3$$

27. If the mean and variance of the data 65, 68, 58, 44, 48, 45, 60, $\alpha, \beta, 60$ where $\alpha > \beta$ are 56 and 66.2 respectively, then $\alpha^2 + \beta^2$ is equal to

Ans. (6344)

Sol. $\bar{x} = 56$

$$\sigma^2 = 66.2$$

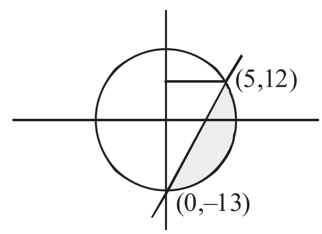
$$\Rightarrow \frac{\alpha^2+\beta^2+25678}{10} - (56)^2 = 66.2$$

$$\therefore \alpha^2 + \beta^2 = 6344$$

28. The area (in sq. units) of the part of circle $x^2 + y^2 = 169$ which is below the line $5x - y = 13$ is $\dfrac{\pi\alpha}{2\beta} - \dfrac{65}{2} + \dfrac{\alpha}{\beta}\sin^{-1}\left(\dfrac{12}{13}\right)$ where α, β are coprime numbers. Then $\alpha + \beta$ is equal to

Ans. (171)

Sol.



$$\text{Area} = \int_{-13}^{12}\sqrt{169-y^2}\,dy - \frac{1}{2}\times 25\times 5$$

$$= \frac{\pi}{2}\times\frac{169}{2} - \frac{65}{2} + \frac{169}{2}\sin^{-1}\frac{12}{13}$$

$$\therefore \alpha + \beta = 171$$

29. If $\dfrac{{}^{11}C_1}{2} + \dfrac{{}^{11}C_2}{3} + ... + \dfrac{{}^{11}C_9}{10} = \dfrac{n}{m}$ with gcd(n, m) = 1, then n + m is equal to

Ans. (2041)

Sol. $\displaystyle\sum_{r=1}^{9}\frac{{}^{11}C_r}{r+1}$

$$= \frac{1}{12}\sum_{r=1}^{9}{}^{12}C_{r+1}$$

$$= \frac{1}{12}\left[2^{12} - 26\right] = \frac{2035}{6}$$

$$\therefore m + n = 2041$$

30. A line with direction ratios 2, 1, 2 meets the lines $x = y + 2 = z$ and $x + 2 = 2y = 2z$ respectively at the point P and Q. if the length of the perpendicular from the point (1, 2, 12) to the line PQ is l, then l^2 is

Ans. (65)

Sol. Let $P(t, t-2, t)$ and $Q(2s-2, s, s)$

D.R's of PQ are 2, 1, 2

$$\frac{2s-2-t}{2} = \frac{s-t+2}{1} = \frac{s-t}{2}$$

$\Rightarrow t = 6$ and $s = 2$

$\Rightarrow P(6,4,6)$ and $Q(2,2,2)$

$$PQ : \frac{x-2}{2} = \frac{y-2}{1} = \frac{z-2}{2} = \lambda$$

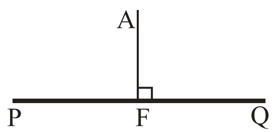
Let $F(2\lambda+2, \lambda+2, 2\lambda+2)$

$A(1,2,12)$

$\overrightarrow{AF} \cdot \overrightarrow{PQ} = 0$

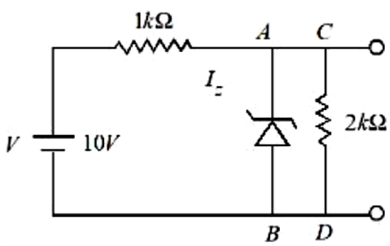
$\therefore \lambda = 2$

So F(6,4, 6) and $AF = \sqrt{65}$



SECTION-A

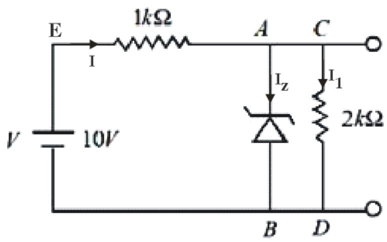
31. In the given circuit, the breakdown voltage of the Zener diode is 3.0 V. What is the value of I_z?



(1) 3.3 mA (2) 5.5 mA

(3) 10 mA (4) 7 mA

Ans. (2)



Sol.

$V_z = 3V$

Let potential at B = 0 V

Potential at E(V_E) = 10 V

$V_C = V_A = 3$ V

$I_z + I_1 = I$

$$I = \frac{10-3}{1000} = \frac{7}{1000} A$$

$$I_1 = \frac{3}{2000} A$$

Therefore $I_z = \dfrac{7-1.5}{1000} = 5.5mA$

32. The electric current through a wire varies with time as $I = I_0 + \beta t$. where $I_0 = 20$ A and $\beta = 3$ A/s. The amount of electric charge crossed through a section of the wire in 20 s is :

(1) 80 C (2) 1000 C

(3) 800 C (4) 1600 C

Ans. (2)

Sol. Given that

Current $I = I_0 + \beta t$

$I_0 = 20A$

$\beta = 3A/s$

$I = 20 + 3t$

$$\frac{dq}{dt} = 20 + 3t$$

$$\int_0^q dq = \int_0^{20} (20+3t)\,dt$$

$$q = \int_0^{20} 20dt + \int_0^{20} 3tdt$$

$$q = \left[20t + \frac{3t^2}{2}\right]_0^{20} = 1000\ C$$

33. Given below are two statements:

Statement I : If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in hot water.

Statement II : If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in cold water.

In the light of the above statements, choose the ***most appropriate*** from the options given below

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Ans. (3)

Sol. Surface tension will be less as temperature increases

$$h = \frac{2T\cos\theta}{\rho gr}$$

Height of capillary rise will be smaller in hot water and larger in cold water.

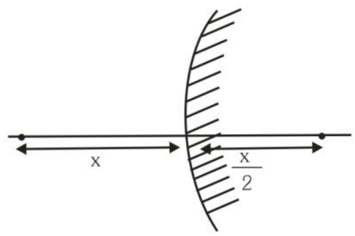
34. A convex mirror of radius of curvature 30 cm forms an image that is half the size of the object. The object distance is :

(1) –15 cm (2) 45 cm

(3) –45cm (4) 15 cm

Ans. (1)

Sol.



Given R = 30 cm

f = R/2 = +15 cm

Magnification (m) $= \pm\frac{1}{2}$

For convex mirror, virtual image is formed for real object.

Therefore, m is +ve

$$\frac{1}{2} = \frac{f}{f-u}$$

u = – 15 cm

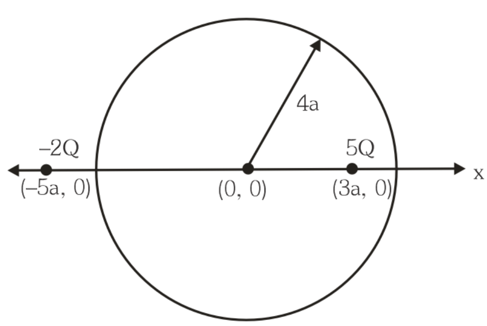
35. Two charges of 5Q and –2Q are situated at the points (3a, 0) and (–5a, 0) respectively. The electric flux through a sphere of radius '4a' having center at origin is :

(1) $\frac{2Q}{\varepsilon_0}$ (2) $\frac{5Q}{\varepsilon_0}$

(3) $\frac{7Q}{\varepsilon_0}$ (4) $\frac{3Q}{\varepsilon_0}$

Ans. (2)

Sol.



5Q charge is inside the spherical region

flux through sphere = $\frac{5Q}{\varepsilon_0}$

36. A body starts moving from rest with constant acceleration covers displacement S_1 in first (p – 1) seconds and S_2 in first p seconds. The displacement $S_1 + S_2$ will be made in time :

(1) $(2p+1)$s

(2) $\sqrt{(2p^2-2p+1)}$s

(3) $(2p-1)$s

(4) $(2p^2-2p+1)$s

Ans. (2)

Sol. S_1 in first (p – 1) sec

S_2 in first p sec

$$S_1 = \frac{1}{2}a(p-1)^2$$

$$S_2 = \frac{1}{2}a(p)^2$$

$$S_1 + S_2 = \frac{1}{2}at^2$$

$(p-1)^2 + p^2 = t^2$

$$t = \sqrt{2p^2+1-2p}$$

37. The potential energy function (in J) of a particle in a region of space is given as U = $(2x^2 + 3y^3 + 2z)$. Here x, y and z are in meter. The magnitude of x - component of force (in N) acting on the particle at point P (1, 2, 3) m is :

(1) 2 (2) 6

(3) 4 (4) 8

Ans. (3)

Sol. Given $U = 2x^2 + 3y^3 + 2z$

$$F_x = -\frac{\partial U}{\partial x} = -4x$$

At x = 1 magnitude of F_x is 4N

38. The resistance $R = \frac{V}{I}$ where $V = (200 \pm 5)$ V and $I = (20 \pm 0.2)$ A, the percentage error in the measurement of R is :

(1) 3.5%

(2) 7%

(3) 3%

(4) 5.5%

Ans. (1)

Sol. $R = \frac{V}{1}$

According to error analysis

$$\frac{dR}{R} = \frac{dV}{V} + \frac{dI}{I}$$

$$\frac{dR}{R} = \frac{5}{200} + \frac{0.2}{20}$$

$$\frac{dR}{R} = \frac{7}{200}$$

% error $\frac{dR}{R} \times 100 = \frac{7}{200} \times 100 = 3.5\%$

39. A block of mass 100 kg slides over a distance of 10 m on a horizontal surface. If the co-efficient of friction between the surfaces is 0.4, then the work done against friction (in J) is :

(1) 4200

(2) 3900

(3) 4000

(4) 4500

Ans. (3)

Sol. Given m = 100 kg

s = 10 m

$\mu = 0.4$

As f = μmg = 0.4 × 100 × 10 = 400 N

Now W = f.s = 400 ×10 = 4000 J

40. Match List I with List II

	List I		List II
A.	$\oint \vec{B}.\vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$	I.	Gauss' law for electricity
B.	$\oint \vec{E}.\vec{dl} = \frac{d\phi_B}{dt}$	II.	Gauss' law for magnetism
C.	$\oint \vec{E}.\vec{dA} = \frac{Q}{\varepsilon_0}$	III.	Faraday law
D.	$\oint \vec{B}.\vec{dA} = 0$	IV.	Ampere – Maxwell law

Chose the correct answer from the options given below

(1) A-IV, B-I, C-III, D-II

(2) A-II, B-III, C-I, D-IV

(3) A-IV, B-III, C-I, D-II

(4) A-I, B-II, C-III, D-IV

Ans. (3)

Sol. Ampere – Maxwell law

$\rightarrow \oint \vec{B}.\vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$

Faraday law $\rightarrow \oint \vec{E}.\vec{dl} = \frac{d\phi_B}{dt}$

Gauss' law for electricity $\rightarrow \oint \vec{E}.\vec{dA} = \frac{Q}{\varepsilon_0}$

Gauss ' law for magnetism $\rightarrow \oint \vec{B}.\vec{dA} = 0$

41. If the radius of curvature of the path of two particles of same mass are in the ratio 3:4, then in order to have constant centripetal force, their velocities will be in the ratio of:

(1) $\sqrt{3} : 2$ (2) $1 : \sqrt{3}$

(3) $\sqrt{3} : 1$ (4) $2 : \sqrt{3}$

Ans. (1)

Sol. Given $m_1 = m_2$

and $\frac{r_1}{r_2} = \frac{3}{4}$

As centripetal force $F = \frac{mv^2}{r}$

In order to have constant (same in this question) centripetal force

$F_1 = F_2$

$$\frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \frac{\sqrt{3}}{2}$$

42. A galvanometer having coil resistance 10 Ω shows a full scale deflection for a current of 3mA. For it to measure a current of 8A, the value of the shunt should be:

(1) $3 \times 10^{-3}\Omega$ (2) $4.85 \times 10^{-3}\Omega$

(3) $3.75 \times 10^{-3}\Omega$ (4) $2.75 \times 10^{-3}\Omega$

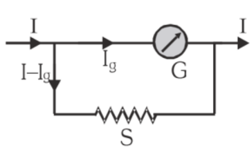
Ans. **(3)**

Sol. Given G = 10 Ω

$I_g = 3mA$

I = 8A

In case of conversion of galvanometer into ammeter.



We have $I_g G = (I - I_g)S$

$$S = \frac{I_g G}{I - I_g}$$

$$S = \frac{(3 \times 10^{-3})10}{8 - 0.003} = 3.75 \times 10^{-3}\Omega$$

43. The de-Broglie wavelength of an electron is the same as that of a photon. If velocity of electron is 25% of the velocity of light, then the ratio of K.E. of electron and K.E. of photon will be:

(1) $\frac{1}{1}$ (2) $\frac{1}{8}$

(3) $\frac{8}{1}$ (4) $\frac{1}{4}$

Ans. **(2)**

Sol. For photon

$$E_P = \frac{hc}{\lambda_p} \Rightarrow \lambda_p = \frac{hc}{E_P}$$

For electron

$$\lambda_e = \frac{h}{m_e v_e} = \frac{h v_e}{2K_e}$$

Given $v_e = 0.25$ c

$$\lambda_e = \frac{h \times 0.25c}{2K_e} = \frac{hc}{8K_e}$$

Also $\lambda_p = \lambda_e$

$$\frac{hc}{E_p} = \frac{hc}{8K_e}$$

$$\frac{K_e}{E_p} = \frac{1}{8}$$

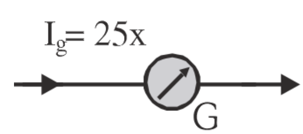
44. The deflection in moving coil galvanometer falls from 25 divisions to 5 division when a shunt of 24Ω is applied. The resistance of galvanometer coil will be :

(1) 12Ω (2) 96Ω

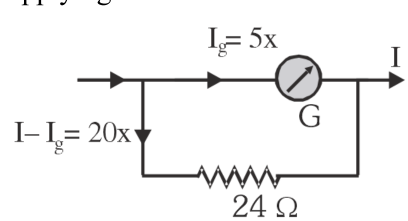
(3) 48Ω (4) 100Ω

Ans. **(2)**

Sol. Let x = current/division



After applying shunt



Now 5x × G = 20x × 24

G = 4 × 24

G = 96Ω

45. A biconvex lens of refractive index 1.5 has a focal length of 20 cm in air. Its focal length when immersed in a liquid of refractive index 1.6 will be:

(1) – 16 cm

(2) – 160 cm

(3) + 160 cm

(4) + 16 cm

Ans. (2)

Sol. $\mu_l = 1.5$

$\mu_m = 1.6$

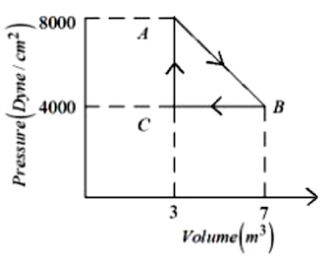
$f_a = 20$ cm

As $\frac{f_m}{f_a} = \frac{(\mu_l - 1)\mu_m}{(\mu_l - \mu_m)}$

$$\frac{f_m}{20} = \frac{(1.5-1)1.6}{(1.5-1.6)}$$

$f_m = -160$ cm

46. A thermodynamic system is taken from an original state A to an intermediate state B by a linear process as shown in the figure. It's volume is then reduced to the original value from B to C by an isobaric process. The total work done by the gas from A to B and B to C would be :

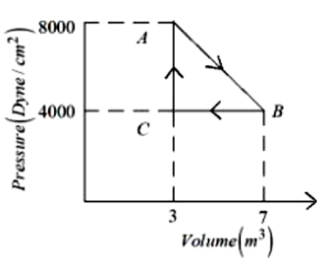


(1) 33800 J (2) 2200 J

(3) 600 J (4) 1200 J

Ans. (BONUS)

Sol.



Work done AB = $\frac{1}{2}$ (8000 + 6000) Dyne/cm^2 × 4m^3 = (6000Dyne/cm^2) × 4m^3

Work done BC = –(4000 Dyne/cm^2) × 4m^3

Total work done = 2000 Dyne/cm^2 × 4m^3

$$= 2\times10^3 \times \frac{1}{10^5}\frac{N}{cm^2}\times 4m^3$$

$$= 2\times10^{-2}\times\frac{N}{10^{-4}m^2}\times 4m^3$$

= 2 × 10^2 × 4 Nm = 800 J

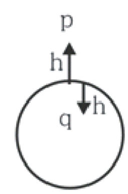
47. At what distance above and below the surface of the earth a body will have same weight, (take radius of earth as R.)

(1) $\sqrt{5}R - R$ (2) $\frac{\sqrt{3}R - R}{2}$

(3) $\frac{R}{2}$ (4) $\frac{\sqrt{5}R - R}{2}$

Ans. (4)

Sol. $g_p = \frac{gR^2}{(R+h)^2}$



$g_q = g\left(1 - \frac{h}{R}\right)$

$g_p = g_q$

$$\frac{g}{\left(1+\frac{h}{R}\right)^2} = g\left(1-\frac{h}{R}\right)$$

$$\left(1-\frac{h^2}{R^2}\right)\left(1+\frac{h}{R}\right) = 1$$

Take $\frac{h}{R} = x$

So

$x^3 - x + x^2 = 0$

$$x = \frac{\sqrt{5}-1}{2}$$

$$h = \frac{R}{2}\left(\sqrt{5}-1\right)$$

48. A capacitor of capacitance 100 μF is charged to a potential of 12 V and connected to a 6.4 mH inductor to produce oscillations. The maximum current in the circuit would be :

(1) 3.2 A (2) 1.5 A

(3) 2.0 A (4) 1.2 A

Ans. **(2)**

Sol. By energy conservation

$$\frac{1}{2}CV^2 = \frac{1}{2}LI_{max}^2$$

$$I_{max} = \sqrt{\frac{C}{L}}\ V$$

$$= \sqrt{\frac{100\times10^{-6}}{6.4\times10^{-3}}}\times12$$

$$= \frac{12}{8} = \frac{3}{2} = 1.5\ \text{A}$$

49. The explosive in a Hydrogen bomb is a mixture of ${}_1H^2$, ${}_1H^3$ and ${}_3Li^6$ in some condensed form. The chain reaction is given by

${}_3Li^6 + {}_0n^1 \rightarrow {}_2He^4 + {}_1H^3$

${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1$

During the explosion the energy released is approximately

[Given : M(Li) = 6.01690 amu. M (${}_1H^2$) = 2.01471 amu. M (${}_2He^4$) = 4.00388amu, and 1 amu = 931.5 MeV]

(1) 28.12 MeV (2) 12.64 MeV

(3) 16.48 MeV (4) 22.22 MeV

Ans. **(4)**

Sol. ${}_3Li^6 + {}_0n^1 \rightarrow {}_2He^4 + {}_1H^3$

${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1$

$${}_3Li^6 + {}_1H^2 \rightarrow 2\left({}_2He^4\right)$$

Energy released in process

$Q = \Delta mc^2$

Q = [M(Li)+ M (${}_1H^2$) –2 × M(${}_2He^4$)] × 931.5 MeV

Q = [6.01690+2.01471–2 × 4.00388] × 931.5 MeV

Q = 22.216 MeV

Q = 22.22 MeV

50. Two vessels A and B are of the same size and are at same temperature. A contains 1g of hydrogen and B contains 1g of oxygen. P_A and P_B are the pressures of the gases in A and B respectively, then $\frac{P_A}{P_B}$ is :

(1) 16 (2) 8 (3) 4 (4) 32

Ans. **(1)**

Sol. $$\frac{P_AV_A}{P_BV_B} = \frac{n_ART_A}{n_BRT_B}$$

Given $V_A = V_B$

And $T_A = T_B$

$$\frac{P_A}{P_B} = \frac{n_A}{n_B}$$

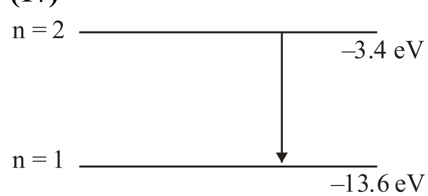
$$\frac{P_A}{P_B} = \frac{1/2}{1/32} = 16$$

SECTION-B

51. When a hydrogen atom going from n = 2 to n = 1 emits a photon, its recoil speed is $\frac{x}{5}$ *m/s*. Where x = ______ . (Use : mass of hydrogen atom = 1.6×10^{-27} kg)

Ans. **(17)**

Sol.



ΔE = 10.2 eV

Recoil speed(v) = $\frac{\Delta E}{mc}$

$$= \frac{10.2\text{eV}}{1.6\times10^{-27}\times3\times10^8}$$

$$= \frac{10.2\times1.6\times10^{-19}}{1.6\times10^{-27}\times3\times10^8}$$

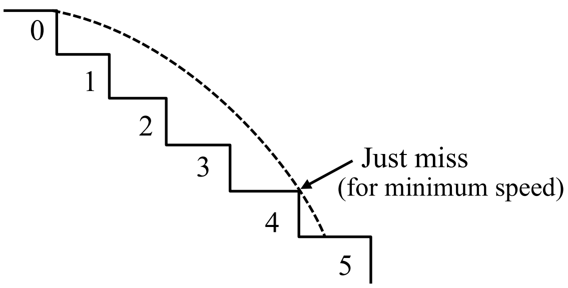
v = 3.4 m/s = $\frac{17}{5}$ m/s

Therefore, x = 17

52. A ball rolls off the top of a stairway with horizontal velocity u. The steps are 0.1 m high and 0.1 m wide. The minimum velocity u with which that ball just hits the step 5 of the stairway will be $\sqrt{x}\ ms^{-1}$ where x = ___________[use g = 10 m/s^2].

Ans. (2)

Sol.



The ball needs to just cross 4 steps to just hit 5th step

Therefore, horizontal range (R) = 0.4 m

R = u.t

Similarly, in vertical direction

$$h=\frac{1}{2}gt^2$$

$$0.4=\frac{1}{2}gt^2$$

$$0.4=\frac{1}{2}g\left(\frac{0.4}{u}\right)^2$$

$u^2 = 2$

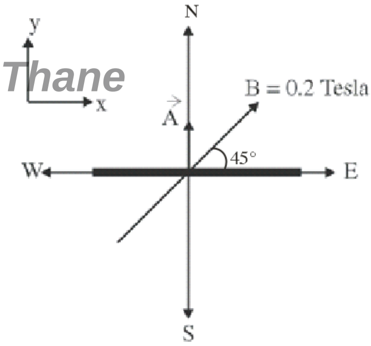
$u = \sqrt{2}$ m/s

Therefore, x = 2

53. A square loop of side 10 cm and resistance 0.7Ω is placed vertically in east-west plane. A uniform magnetic field of 0.20 T is set up across the plane in north east direction. The magnetic field is decreased to zero in 1 s at a steady rate. Then, magnitude of induced emf is $\sqrt{x}\times 10^{-3}$V. The value of x is ________.

Ans. (2)

Sol.



$$\vec{A}=(0.1)^2\,\hat{j}$$

$$\vec{B}=\frac{0.2}{\sqrt{2}}\hat{i}+\frac{0.2}{\sqrt{2}}\hat{j}$$

Magnitude of induced emf

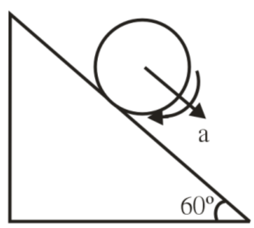
$$e=\frac{\Delta\phi}{\Delta t}=\frac{\vec{B}\cdot\vec{A}-0}{1}=\sqrt{2}\times 10^{-3}\,V$$

54. A cylinder is rolling down on an inclined plane of inclination 60°. It's acceleration during rolling down will be $\frac{x}{\sqrt{3}}\,m/s^2$, where x = _________.

(use g = 10 m/s^2).

Ans. (10)

Sol.



For rolling motion, $a=\dfrac{g\sin\theta}{1+\dfrac{I_{cm}}{MR^2}}$

$$a=\frac{g\sin\theta}{1+\frac{1}{2}}$$

$$=\frac{2\times 10\times\frac{\sqrt{3}}{2}}{3}$$

$$=\frac{10}{\sqrt{3}}$$

Therefore x = 10

55. The magnetic potential due to a magnetic dipole at a point on its axis situated at a distance of 20 cm from its center is 1.5×10^{-5} Tm. The magnetic moment of the dipole is ________ Am^2.

(Given : $\frac{\mu_0}{4\pi} = 10^{-7} TmA^{-1}$)

Ans. **(6)**

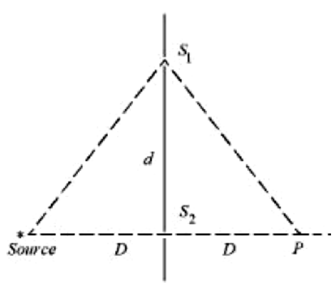
Sol. $V = \frac{\mu_0}{4\pi}\frac{M}{r^2}$

$$\Rightarrow 1.5\times10^{-5} = 10^{-7} \times \frac{M}{\left(20\times10^{-2}\right)^2}$$

$$\Rightarrow M = \frac{1.5\times10^{-5}\times20\times20\times10^{-4}}{10^{-7}}$$

$M = 1.5 \times 4 = 6$

56. In a double slit experiment shown in figure, when light of wavelength 400 nm is used, dark fringe is observed at P. If D = 0.2 m. the minimum distance between the slits S_1 and S_2 is ______ mm.



Ans. **(0.20)**

Sol. Path difference for minima at P

$$2\sqrt{D^2+d^2} - 2D = \frac{\lambda}{2}$$

$$\therefore \sqrt{D^2+d^2} - D = \frac{\lambda}{4}$$

$$\therefore \sqrt{D^2+d^2} = \frac{\lambda}{4} + D$$

$$\Rightarrow D^2 + d^2 = D^2 + \frac{\lambda^2}{16} + \frac{D\lambda}{2}$$

$$\Rightarrow d^2 = \frac{D\lambda}{2} + \frac{\lambda^2}{16}$$

$$\Rightarrow d^2 = \frac{0.2\times400\times10^{-9}}{2} + \frac{4\times10^{-14}}{4}$$

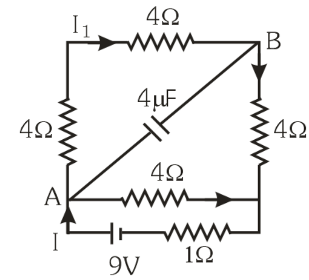
$$\Rightarrow d^2 \approx 400\times10^{-10}$$

$$\therefore d = 20\times10^{-5}$$

$$\Rightarrow d = 0.20\,mm$$

57. A 16Ω wire is bend to form a square loop. A 9V battery with internal resistance 1Ω is connected across one of its sides. If a 4μF capacitor is connected across one of its diagonals, the energy stored by the capacitor will be $\frac{x}{2}$ μJ. where x = ________.

Ans. **(81)**



Sol.

$$I = \frac{V}{R_{eq}} \quad I = \frac{V}{R_{eq}} = \frac{9}{1+\frac{12\times4}{12+4}} = \frac{9}{4}$$

$$I_1 = \frac{9}{4}\times\frac{4}{16} = \frac{9}{16}$$

$$V_A - V_B = I_1 \times 8 = \frac{9}{16}\times8 = \frac{9}{2}\ \mathbf{V}$$

$$\therefore U = \frac{1}{2}\times4\times\frac{81}{4}\mu J$$

$$\therefore U = \frac{81}{2}\mu J$$

$$\therefore x = 81$$

58. When the displacement of a simple harmonic oscillator is one third of its amplitude, the ratio of total energy to the kinetic energy is $\frac{x}{8}$, where x = ________.

Ans. **(9)**

Sol. Let total energy = E = $\frac{1}{2}KA^2$

$$U = \frac{1}{2}K\left(\frac{A}{3}\right)^2 = \frac{KA^2}{2\times9} = \frac{E}{9}$$

$$KE = E - \frac{E}{9} = \frac{8E}{9}$$

$$\text{Ratio } \frac{\text{Total}}{KE} = \frac{E}{\frac{8E}{9}} = \frac{9}{8}$$

x = 9

59. An electron is moving under the influence of the electric field of a uniformly charged infinite plane sheet S having surface charge density $+\sigma$. The electron at $t = 0$ is at a distance of 1 m from S and has a speed of 1 m/s. The maximum value of σ if the electron strikes S at $t = 1$ s is $\alpha\left[\frac{m \in_0}{e}\right]\frac{C}{m^2}$ the value of α is

Ans. (8)

Sol. $u = 1\ m/s;\ a = -\frac{\sigma e}{2\varepsilon_0 m}$

$t = 1\ s$

$S = -1\ m$

Using $S = ut + \frac{1}{2}at^2$

$-1 = 1\times1 - \frac{1}{2}\times\frac{\sigma e}{2\varepsilon_0 m}\times(1)^2$

$\therefore \sigma = 8\frac{\varepsilon_0 m}{e}$

$\therefore \alpha = 8$

60. In a test experiment on a model aeroplane in wind tunnel, the flow speeds on the upper and lower surfaces of the wings are 70 ms^{-1} and 65 ms^{-1} respectively. If the wing area is 2 m^2 the lift of the wing is ________ N.

(Given density of air = 1.2 kg m^{-3})

Ans. (810)

Sol. $F = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)A$

$F = \frac{1}{2}\times1.2\times\left(70^2 - 65^2\right)\times2$

$= 810$ N

SECTION-A

61. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**:

Assertion A: The first ionisation enthalpy decreases across a period.

Reason R: The increasing nuclear charge outweighs the shielding across the period.

In the light of the above statements, choose the most appropriate from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) A is true but R is false

(3) A is false but R is true

(4) Both A and R are true but R is NOT the correct explanation of A

Ans. (3)

Sol. First ionisation energy **increases** along the period.

Along the period Z increases which outweighs the shielding effect

62. Match List I with List II

LIST-I (Substances)	**LIST-II (Element Present)**
A.Ziegler catalyst	I.Rhodium
B.Blood Pigment	II. Cobalt
C.Wilkinson catalyst	III.Iron
D.Vitamin B_{12}	IV.Titanium

Choose the correct answer from the options given below:

(1) A-II, B-IV, C-I, D-III

(2) A-II, B-III, C-IV, D-I

(3) A-III, B-II, C-IV, D-I

(4) A-IV, B-III, C-I, D-II

Ans. (4)

Sol. Ziegler catalyst $\rightarrow$ Titanium

Blood pigment $\rightarrow$ Iron

Wilkinson catalyst $\rightarrow$ Rhodium

Vitamin $B_{12} \rightarrow$ Cobalt

63. In chromyl chloride test for confirmation of Cl^- ion, a yellow solution is obtained. Acidification of the solution and addition of amyl alcohol and 10% H_2O_2 turns organic layer blue indicating formation of chromium pentoxide. The oxidation state of chromium in that is

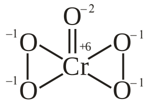
(1)+6 (2)+5

(3)+10 (4)+3

Ans. (1)

Sol. $Cl^- + K_2Cr_2O_7 + H_2SO_4 \rightarrow CrO_2Cl_2 \xrightarrow{\text{Basic medium}} \underset{\text{yellow solution}}{CrO_4^{2-}} + Cl^-$

$$\underset{\text{yellow solution}}{CrO_4^{2-}} \xrightarrow[\text{3. 10\% } H_2O_2]{\text{1. Acidification, 2. Amyl alcohol}} \underset{\text{blue compound}}{CrO_5}$$



64. The difference in energy between the actual structure and the lowest energy resonance structure for the given compound is

(1) electromeric energy

(2) resonance energy

(3) ionization energy

(4) hyperconjugation energy

Ans. (2)

Sol. The difference in energy between the actual structure and the lowest energy resonance structure for the given compound is known as resonance energy.

65. Given below are two statements :

Statement I : The electronegativity of group 14 elements from Si to Pb gradually decreases.

Statement II : Group 14 contains non-metallic, metallic, as well as metalloid elements.

In the light of the above statements, choose the most appropriate from the options given below :

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Ans. (1)

Sol.

Gr-14	EN
C	2.5
Si	1.8
Ge	1.8
Sn	1.8
Pb	1.9

The electronegativity values for elements from Si to Pb are almost same. So Statement I is false.

66. The correct set of four quantum numbers for the valence electron of rubidium atom (Z = 37) is:

(1) $5, 0, 0, +\frac{1}{2}$ (2) $5, 0, 1, +\frac{1}{2}$

(3) $5, 1, 0, +\frac{1}{2}$ (4) $5, 1, 1, +\frac{1}{2}$

Ans. (1)

Sol. $Rb = [Kr]5s^1$

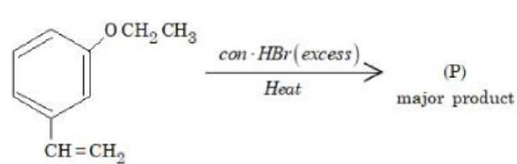
n = 5

$l = 0$

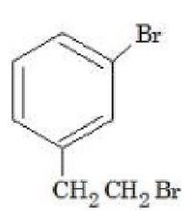
m = 0

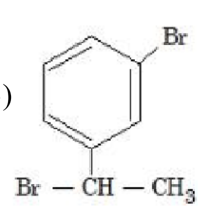
s = +½ or –½

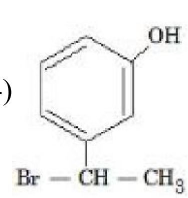
67. The major product(P) in the following reaction is



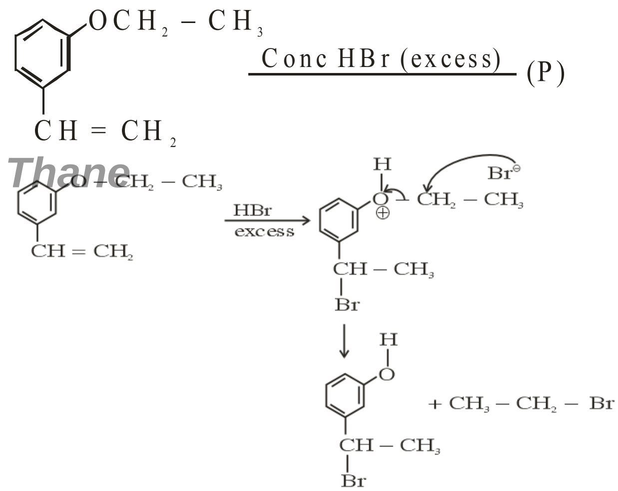
(1) 

(2) 

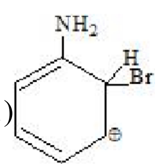
(3) 

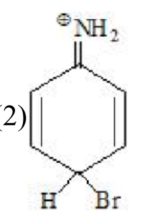
(4) 

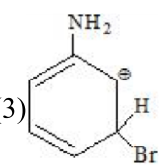
Ans. (4)

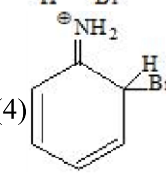
Sol. 

68. The arenium ion which is not involved in the bromination of Aniline is .

(1) 

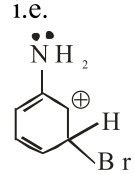
(2) 

(3) 

(4) 

Ans. (3)

Sol. Since $-\ddot{N}H_2$ group is o/p directing hence arenium ion will not be formed by attack at meta position i.e.



Hence Answer is (3)

69. Appearance of blood red colour, on treatment of the sodium fusion extract of an organic compound with $FeSO_4$ in presence of concentrated H_2SO_4 indicates the presence of element/s

(1) Br (2) N

(3) N and S (4) S

Ans. (3)

Sol. $Fe^{2+} \xrightarrow[\text{Conc.}H_2SO_4]{H^+} Fe^{+3}$

$Fe^{+3} \xrightarrow{^-SCN} Fe(SCN)_3$ (blood red colour)

Appearance of blood red colour indicates presence of both nitrogen and sulphur.

70. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : Aryl halides cannot be prepared by replacement of hydroxyl group of phenol by halogen atom.

Reason R : Phenols react with halogen acids violently.

In the light of the above statements, choose the most appropriate from the options given below:

(1) Both A and R are true but R is NOT the correct explanation of A

(2) A is false but R is true

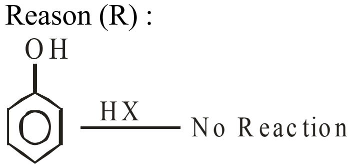
(3) A is true but R is false

(4) Both A and R are true and R is the correct explanation of A

Ans. (3)

Sol. Assertion (A): Given statement is correct because in phenol hydroxyl group cannot be replaced by halogen atom.

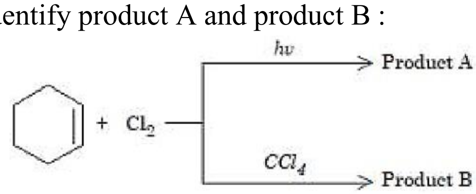
Reason (R) :

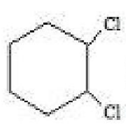
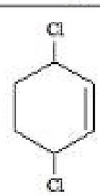


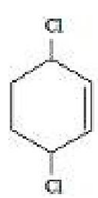
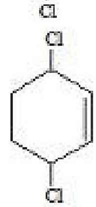
Given reason is false

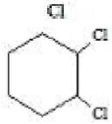
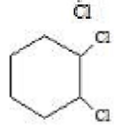
Hence Assertion (A) is correct but Reason (R) is false

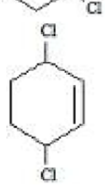
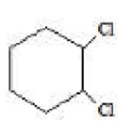
71. Identify product A and product B :



(1) A :  B : 

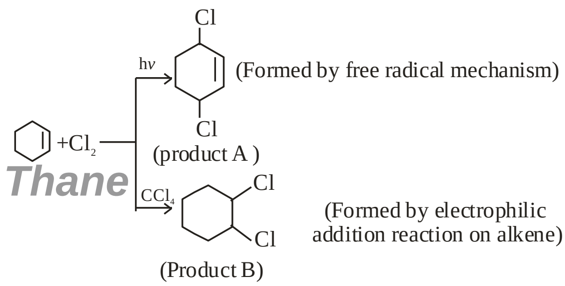
(2) A :  B : 

(3) A :  B : 

(4) A :  B : 

Ans. (4)

Sol.



Hence correct Ans. (4)

72. Identify the incorrect pair from the following :

(1) Fluorspar- BF_3

(2) Cryolite-Na_3AlF_6

(3) Fluoroapatite-$3Ca_3(PO_4)_2.CaF_2$

(4) Carnallite-$KCl.MgCl_2.6H_2O$

Ans. (1)

Sol. (1) Fluorspar is CaF_2

73. The interaction between π bond and lone pair of electrons present on an adjacent atom is responsible for

(1) Hyperconjugation

(2) Inductive effect

(3) Electromeric effect

(4) Resonance effect

Ans. (4)

Sol. It is a type of conjugation responsible for resonance.

74. $KMnO_4$ decomposes on heating at 513K to form O_2 along with

(1) MnO_2 & K_2O_2

(2) K_2MnO_4 & Mn

(3) Mn & KO_2

(4) K_2MnO_4 & MnO_2

Ans. (4)

Sol. $KMnO_4 \xrightarrow{\Delta} K_2MnO_4 + MnO_2 + O_2$

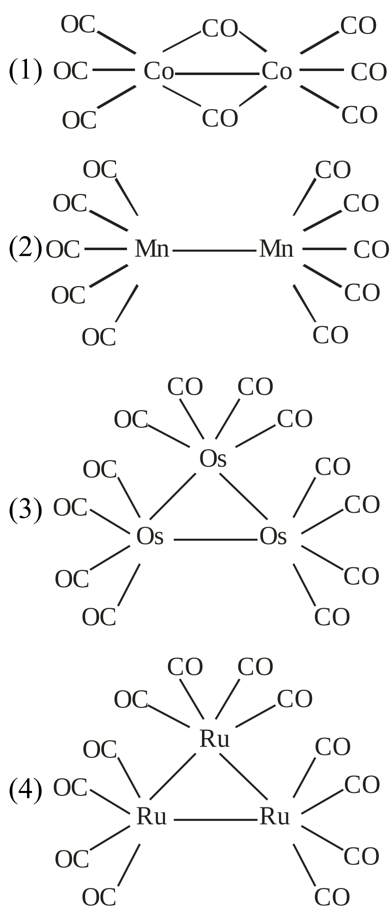
75. In which one of the following metal carbonyls, CO forms a bridge between metal atoms?

(1) $[Co_2(CO)_8]$ (2) $[Mn_2(CO)_{10}]$

(3) $[Os_3(CO)_{12}]$ (4) $[Ru_3(CO)_{12}]$

Ans. (1)

Sol.



76. Type of amino acids obtained by hydrolysis of proteins is :

(1) β (2) α

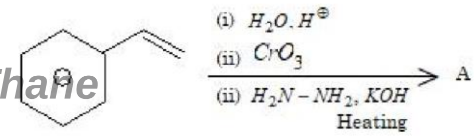
(3) δ (4) γ

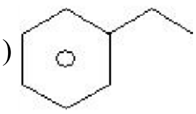
Ans. (2)

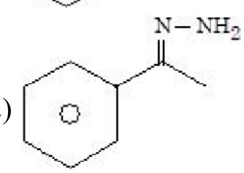
Sol. Proteins are natural polymers composed of α-amino acids which are connected by peptide linkages.

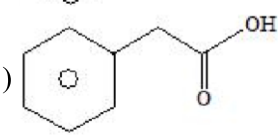
Hence proteins upon acidic hydrolysis produce α-amino acids.

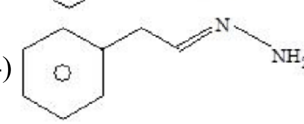
77. The final product A formed in the following multistep reaction sequence is



(1) 

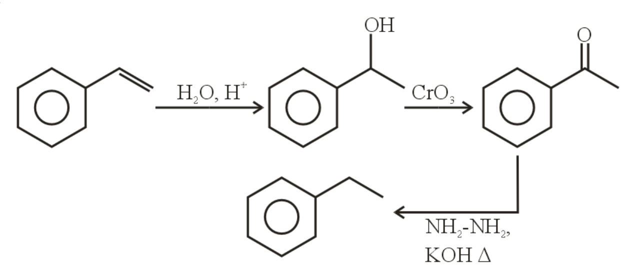
(2) 

(3) 

(4) 

Ans. (1)

Sol.



78. Which of the following is **not** correct?

(1) ΔG is negative for a spontaneous reaction

(2) ΔG is positive for a spontaneous reaction

(3) ΔG is zero for a reversible reaction

(4) ΔG is positive for a non-spontaneous reaction

Ans. (2)

Sol. $(\Delta G)_{P,T} = (+)$ve for non-spontaneous process

79. Chlorine undergoes disproportionation in alkaline medium as shown below :

a $Cl_2(g)$ + b $OH^-(aq) \rightarrow$ c $ClO^-(aq)$ + d $Cl^-(aq)$ + e $H_2O(l)$

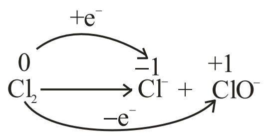
The values of a, b, c and d in a balanced redox reaction are respectively :

(1) 1, 2, 1 and 1 (2) 2, 2, 1 and 3

(3) 3, 4, 4 and 2 (4) 2, 4, 1 and 3

Ans. (1)

Sol.



$$\Rightarrow Cl_2 + 2\bar{O}H \longrightarrow Cl^- + ClO^- + H_2O$$

80. In alkaline medium. MnO_4^- oxidises I^- to

(1) IO_4^- (2) IO^-

(3) I_2 (4) IO_3^-

Ans. (4)

Sol. $2MnO_4^- + H_2O + I^- \xrightarrow{\text{alkaline medium}} 2MnO_2 + 2OH^- + IO_3^-$

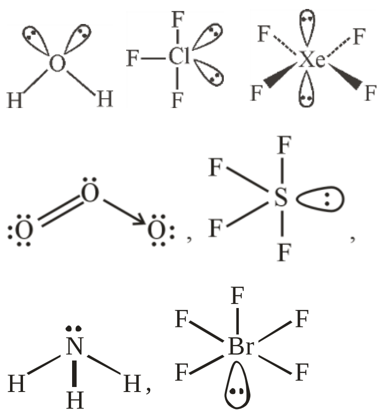
SECTION-B

81. Number of compounds with one lone pair of electrons on central atom amongst following is _

O_3, H_2O, SF_4, ClF_3, NH_3, BrF_5, XeF_4

Ans. (4)

Sol.



82. The mass of zinc produced by the electrolysis of zinc sulphate solution with a steady current of 0.015 A for 15 minutes is____ $\times 10^{-4}$ g.

(Atomic mass of zinc = 65.4 amu)

Ans. (45.75) or (46)

Sol. $Zn^{+2} + 2e^- \longrightarrow Zn$

$W = Z \times i \times t$

$$= \frac{65.4}{2 \times 96500} \times 0.015 \times 15 \times 60$$

$$= 45 \cdot 75 \times 10^{-4}\,\text{gm}$$

83. For a reaction taking place in three steps at same temperature, overall rate constant $K = \frac{K_1K_2}{K_3}$. If Ea_1, Ea_2 and Ea_3 are 40, 50 and 60 kJ/mol respectively, the overall Ea is____kJ/mol.

Ans. (30)

Sol. $K = \frac{K_1 \cdot K_2}{K_3} = \frac{A_1 \cdot A_2}{A_3} \cdot e^{-\frac{(E_{a_1} + E_{a_2} - E_{a_3})}{RT}}$

$$A \cdot e^{-E_a/RT} = \frac{A_1A_2}{A_3} \cdot e^{-\frac{(E_{a_1} + E_{a_2} - E_{a_3})}{RT}}$$

$$E_a = E_{a_1} + E_{a_2} - E_{a_3} = 40 + 50 - 60 = 30 \text{ kJ / mole.}$$

84. For the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$, K_p= 0.492 atm at 300K. K_c for the reaction at same temperature is___ $\times 10^{-2}$.

(Given : R = 0.082 L atm mol^{-1} K^{-1})

Ans. (2)

Sol. $K_P = K_C \cdot (RT)^{\Delta n_g}$

$\Delta n_g = 1$

$$\Rightarrow K_c = \frac{K_P}{RT} = \frac{0.492}{0.082 \times 300} = 2 \times 10^{-2}$$

85. A solution of H_2SO_4 is 31.4% H_2SO_4 by mass and has a density of 1.25g/mL.The molarity of the H_2SO_4 solution is____M (nearest integer)

[Given molar mass of H_2SO_4 = 98g mol^{-1}]

Ans. (4)

Sol. $M = \frac{n_{solute}}{V} \times 1000$

$$= \frac{\left(\frac{31.4}{98}\right)}{\left(\frac{100}{1.25}\right)} \times 1000$$

$$= 4.005 \approx 4$$

86. The osmotic pressure of a dilute solution is 7×10^5 Pa at 273K. Osmotic pressure of the same solution at 283K is____$\times 10^4$ Nm^{-2}.

Ans. (72.56) or (73)

Sol. $\pi = CRT$

$$\Rightarrow \frac{\pi_1}{\pi_2} = \frac{T_1}{T_2}$$

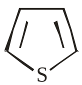
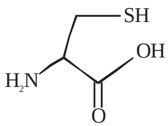
$$\Rightarrow \pi_2 = \frac{\pi_1 T_2}{T_1} = \frac{7 \times 10^5 \times 283}{273}$$

$$= 72.56 \times 10^4\ Nm^{-2}$$

87. Number of compounds among the following which contain sulphur as heteroatom is____.

Furan,Thiophene, Pyridine, Pyrrole, Cysteine, Tyrosine

Ans. (2)

Sol.  , 

Thiophene

Cysteine

88. The number of species from the following which are paramagnetic and with bond order equal to one is____.

$H_2, He_2^+, O_2^+, N_2^{2-}, O_2^{2-}, F_2, Ne_2^+, B_2$

Ans. (1)

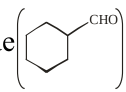
Sol.

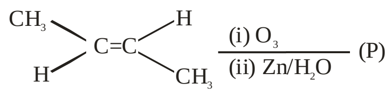
	Magnetic behaviour	Bond order
H_2	Diamagnetic	1
He_2^+	Paramagnetic	0.5
O_2^+	Paramagnetic	2.5
N_2^{2-}	Paramagnetic	2
O_2^{2-}	Diamagnetic	1
F_2	Diamagnetic	1
Ne_2^+	Paramagnetic	0.5
B_2	Paramagnetic	1

89. From the compounds given below, number of compounds which give positive Fehling's test is____.

Benzaldehyde, Acetaldehyde, Acetone, Acetophenone,Methanal, 4-nitrobenzaldehyde, cyclohexane carbaldehyde.

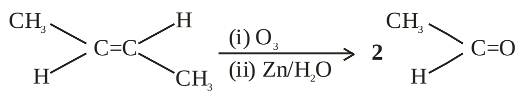
Ans. (3)

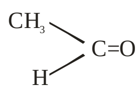
Sol. Acetaldehyde (CH_3CHO),Methanal(HCHO), and cyclohexane carbaldehyde.

90. 

Consider the given reaction. The total number of oxygen atoms present per molecule of the product (P) is____.

Ans. (1)

Sol. 

Hence total number of oxygen atom present per molecule  is 1.